

Gravitational wave extraction from an inspiraling configuration of merging black holes

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We present new techniques for evolving binary black hole systems which allow the accurate determination of gravitational waveforms directly from the wave zone region of the numerical simulations. Rather than excising the black hole interiors, our approach follows the “puncture” treatment of black holes, but utilizing a new gauge condition which allows the black holes to move successfully through the computational domain. We apply these techniques to an inspiraling binary, modeling the radiation generated during the final plunge and ringdown. We demonstrate convergence of the waveforms and good conservation of mass-energy, with just over 3% of the system’s mass converted to gravitational radiation.

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Coalescing comparable mass black hole binaries are prodigious sources of gravitational waves. The final merger of these systems, in which the black holes leave their quasicircular orbits and plunge together to produce a highly distorted black hole that “rings down” to a quiescent Kerr state, will produce a strong burst of gravitational radiation. Such mergers are expected to be among the strongest sources for ground-based gravitational wave detectors, which will observe the mergers of stellar-mass and intermediate mass black hole binaries, and the space-based LISA, which will detect mergers of massive black hole binaries. Observations of these systems will provide an unprecedented look into the strong-field dynamical regime of general relativity. With the first-generation of ground-based interferometers reaching maturity and LISA moving forward through the formulation phase, the need for accurate merger waveforms has become urgent.

Such waveforms can only be obtained through 3-D numerical relativity simulations of the full Einstein equations. While this has proven to be a very challenging undertaking, new developments allow an optimistic outlook. Full 3-D evolutions of binary black holes, in which regions within the horizons have been excised from the computational grid, have recently been carried out. Using co-rotating coordinates, so that the holes remain fixed on the grid as the system evolves, a binary has been evolved through a little more than a full orbit [1] as well as through a plunge, merger, and ringdown [2], though without being able to extract gravitational waveforms. More recently, a simulation in which excised black holes move through the grid in a single plunge-orbit, merger, and ringdown has been accomplished, with the calculation of a waveform [3].

In this *Letter*, we report the results of new simulations of inspiraling binary black holes through merger and ringdown. These have been carried out using new techniques which allow the black holes to move through the coordinate grid without the need for excision [17]. Using fixed mesh refinement, we are able to resolve both the dynamical region where the black holes inspiral (with

length scales $\sim M$, where M is the total system mass) and the outer regions where the gravitational waves propagate (length scales $\sim (10 - 100)M$). Using an outer boundary at $128M$, we evolve the system to well beyond $t \approx 100M$, extract gravitational waveforms and demonstrate that they are 2nd-order convergent.

We start by setting up “puncture” initial data for equal mass binary black holes [4]. The metric on the initial spacelike slice takes the form $g_{ij} = \psi^4 \delta_{ij}$, where $i, j = 1, 2, 3$, and the conformal factor $\psi = \psi_{\text{BL}} + u$. The static, singular part of the conformal factor has the form $\psi_{\text{BL}} = 1 + \sum_{n=1}^2 m_n / 2|\vec{r} - \vec{r}_n|$, where the n^{th} black hole has mass m_n and its located at \vec{r}_n . The nonsingular function u is obtained by solving the Hamiltonian constraint equation using AMRMG [5]. We use parameters so that the black holes have proper separation $4.99M$, and the system has total mass $M = 1.008$ and angular momentum $J = 0.779M^2$. This corresponds to the run QC0 studied in Ref. [6].

In the standard puncture implementation, ψ_{BL} is factored out and handled analytically; only the regular parts of the metric are evolved. In this case, the punctures remain fixed on the grid while the binary evolves. However, the stretching of the coordinate system that ensues is problematical. First, as the physical distance between the black holes shrinks, certain components of the metric must approach zero, causing other quantities to grow uncontrollably. Second, a corotating coordinate frame (implemented by an appropriate angular shift vector) is necessary to keep the orbiting punctures fixed on the grid; this causes extremely superluminal coordinate speeds at large distances from the black holes and, in the case of a Cartesian grid, incoming noise from the outer boundary.

Our approach is to allow the punctures to move freely through the grid, by not factoring out the singular part of the conformal factor but rather evolving it inseparably from the regular part. Initially, we follow the standard puncture technique and set up the binary so that the centers of the black holes are not located at a grid point. Taking numerical derivatives of ψ_{BL} effectively

regularizes the puncture singularity using the smoothing inherent in the finite differences. These regularized data are then evolved numerically. Since the centers of the black holes remain in the $z = 0$ plane, they do not pass through gridpoints in our cell-centered implementation.

We evolve this data with the *Hahndol* code, which uses a conformal formulation of Einstein’s evolution equations on a cell-centered numerical grid [7] with a box-in-box resolution structure implemented via *Paramesh* [8]. The innermost refinement region is a cube stretching from $-2M$ to $2M$ in all 3 dimensions, and has the finest resolution h_f . The punctures are placed within this region on the y -axis in the $z = 0$ plane; we impose equatorial symmetry throughout. We performed three simulations with identical grid structures, but with uniformly differing resolutions. In the most refined cubical region the resolutions were $h_f = M/16$, $M/24$, and $M/32$. Subsequent boxes of doubled size have half the resolution. We use 8 boxes to put the outer boundary at $128M$, causally disconnected from the wave extraction region through most of the run. We use 4th order finite differencing for the spatial derivatives except for the advection of the shift, which is performed with 2nd-order, mesh-adapted differencing [9], and we use 2nd-order time stepping via a three-step iterative Crank-Nicholson scheme.

In our new approach, the free evolution of punctures is made possible by a modified version of a common coordinate condition known as the Gamma-freezing shift vector, which drives the coordinates towards quiescence as the merged remnant black hole also becomes physically quiescent. Our modified version retains this “freezing” property, yet is suitable for motile punctures. Specifically we use $\partial_t \beta^i = \frac{3}{4} \alpha B^i$ and $\partial_t B^i = \partial_t \tilde{\Gamma}^i - \beta^j \partial_j \tilde{\Gamma}^i - \eta B^i$, which incorporates two critical changes to the standard Gamma-freezing condition. A factor ψ_{BL} of the conformal factor, originally used to ensure that the shift vanishes at the puncture, has been removed in order to allow the punctures to move. Also, a new term has been added ($-\beta^j \partial_j \tilde{\Gamma}^i$) which facilitates more stable and accurate evolution of moving punctures by eliminating a zero-speed mode (which was otherwise found to create a “puncture memory” effect as errors grew in place [10]). Along with this shift condition, we use the standard singularity-avoiding, 1+log slicing condition on the lapse.

Fig. 1 shows the error in the Hamiltonian constraint C_H at 2 different times. The peak violation near the punctures does not leak-out, or grow with time, but stays well-confined even though the punctures and horizons move across the grid. Overall, we get 2nd-order convergence away from the horizons to well beyond the wave extraction region for the entire course of the run. There is no indication of exponentially growing constraint violations which have plagued many numerical simulations with black holes fixed in place on the numerical grid.

One way to get a picture of the motion of the black holes is to look at the location of the black hole appar-

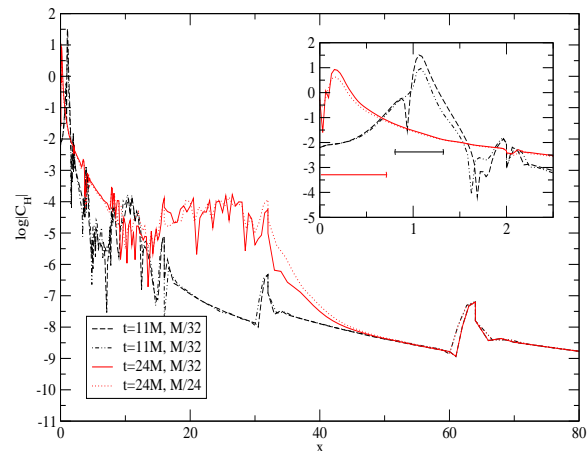


FIG. 1: Hamiltonian constraint error C_H for $h_f = M/24$ and $M/32$, at two times when a puncture is near to crossing the positive x -axis. The data are scaled such that the lines should superpose in the case of perfect 2nd-order convergence. The inset shows that C_H is well-behaved in the region near the punctures. The horizontal lines indicate the approximate location of the apparent horizons; at the later time a common horizon has formed.

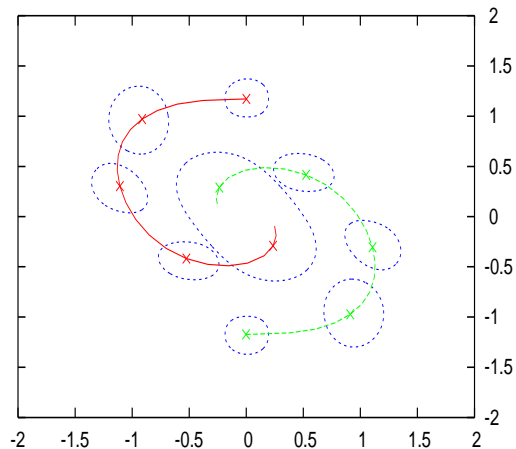


FIG. 2: The positions of the apparent horizons at times $t = 0, 5, 10, 15,$ and $20M$ for our $M/16$ run. The curve shows the trajectories of centroids of the individual apparent horizons.

ent horizons at different times. Fig. 2 shows the locations of a sequence of apparent horizons (calculated using the *AHFINDERDIRECT* code[11]) where they cross the x - y -plane for our $h_f = M/16$ run. In the coordinates of our simulation, the black holes undergo about one-half orbit before forming a common horizon.

We extract the gravitational waves generated by the merger using the technique explained in detail in [12]. Fig. 3 shows the dominant $l = 2, m = 2$ components of the Weyl curvature scalar Ψ_4 extracted at 2 different radii from the medium and high resolution runs. For each resolution, the time-shifted and rescaled waveforms computed at different extraction radii are nearly indistin-

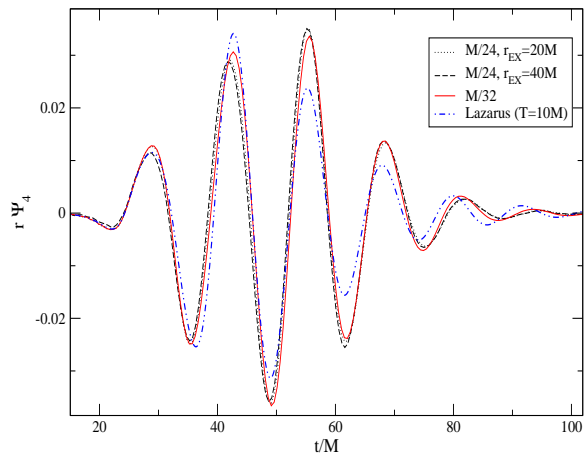


FIG. 3: Real part of $r\Psi_4$ extracted from the numerical simulation on spheres of radii $r_{EX} = 20$, and $40M$ for the medium and high resolution runs. The waveforms extracted at different radii have been rescaled by $1/r_{EX}$ and shifted in time to account for the wave propagation time between the extraction spheres. At high resolution ($h_f = M/32$) there is no discernible dependence on extraction radius. For comparison, we show Lazarus waveforms from Ref. [6].

guisable, indicating that the waves travel cleanly across refinement boundaries and have the expected $1/r$ falloff. In addition, the two highest resolution waveforms differ only by a slight phase shift, and a by few percent in amplitude. For comparison we have also included the QC0 Lazarus waveforms from Sec. V of Ref. [6]. These were extracted by approximately matching the later portion of brief numerical simulations onto a perturbed black hole [13, 14] at transition time $10M$.

Fig. 4 shows the convergence of the extracted waves throughout the run. The difference between the two highest resolutions is roughly 90 degrees out of phase with the waveform, corresponding to a small phase-shift in the waveform, possibly caused by a small difference in the orbital trajectories.

The gravitational waveforms also contain physical information about the radiation, including the energy E and angular momentum J carried away by the radiation. We calculate dE/dt and dJ/dt from time integrals of all $l = 2$ waveform components using Eqs. (5.1) and (5.2) in [6]. Integrating dE/dt gives the energy loss as a function of time; this should be the same as $M - M_{ADM}$, where M_{ADM} is the ADM mass extracted at on a sphere of sufficiently large radius [15]. Fig. 5 shows a comparison between $M - E$ and an independent calculation of M_{ADM} at two extraction radii $r_{EX} = 40$ and $50M$. The striking consistency between the two calculations as the radiation passes indicates good energy conservation in the simulation. Shortly before the arrival of the radiation the ADM mass measurement is affected by a transient non-physical pulse in the gauge evolution, though the pulse does not affect the radiation measurement.

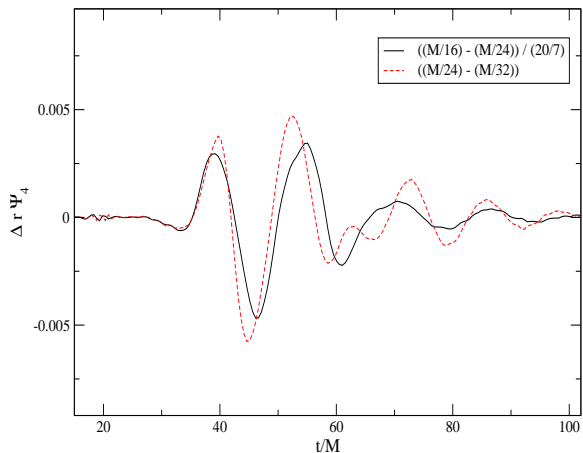


FIG. 4: Differences of the real part of $r\Psi_4$ for resolutions of $h_f = M/16, M/24$, and $M/32$ appropriately scaled such that for perfect 2nd-order convergence the lines would lay on top of each other.

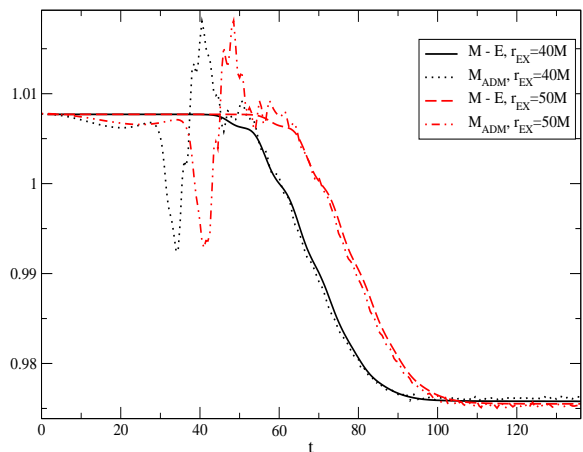


FIG. 5: Conservation of mass-energy for the highest resolution case, $h_f = M/32$. We compare the ADM mass M_{ADM} with the mass remaining, $M - E$, after gravitational radiation energy loss E . The good agreement, based on extraction spheres at $r_{EX} = 40$ and $50M$, indicates conservation of energy in the simulation.

The total radiated energy calculated from the waveforms extracted at $r_{EX} = 20, 30, 40$ and $50M$ in the highest resolution run has the values $E/M = 0.0304, 0.0312, 0.0317$ and 0.0319 , respectively. While these values vary significantly with r_{EX} (even extracting at these relatively large radii), they are neatly consistent with a $1/r_{EX}$ falloff to an asymptotic value of 0.0330 with an uncertainty in the extrapolation of $< 1\%$. In Table I we give the total radiated energies and angular momenta extrapolated as $r_{EX} \rightarrow \infty$. For comparison we also include the Lazarus values, as well as values from the AEI group [2] which did not determine waveforms, but estimated the radiative losses based on the state of the final black hole horizon in runs including the QC0 case. Our lowest res-

	$M/16$	$M/24$	$M/32$	Lazarus	AEI
E/M	0.0516	0.0342	0.0330	0.025	0.030
J/M^2	0.208	0.140	0.138	0.10	0.17

TABLE I: The radiated energy E and angular momentum J carried away by gravitational radiation in our simulations. Our values are comparable with earlier estimates from the AEI (via horizon analysis) and Lazarus via perturbation techniques.

olution run clearly over-estimates the radiation energy and angular momentum while our higher resolution results are in closer agreement with the AEI value for the energy, and close to the 20% level of confidence suggested in Ref. [6].

In conclusion, we have calculated a gravitational radiation waveform directly via numerical simulation of an inspiraling configuration of binary black holes. Starting with equal mass puncture black holes and parameters corresponding to the QC0 model of Ref. [6], we release the holes and allow them to move through the grid without excision. A new gauge condition is used that allows us to accurately evolve this system from the initial inspiral orbit through merger and ringdown. The simulations converge with increasing resolution to 2nd-order, leading to a 2nd-order convergence of the waveform. These waveforms have the correct $1/r$ falloff and agree to a great extent with approximatively calculated ones. Our simulations show good energy conservation as indicated by comparing the change in ADM mass with the radiated energy. The QC0 configuration provides a model for the final plunge of the two black holes and the subsequent ringdown. In this brief burst of gravitational radiation we find that just over 3% of the system's initial mass-energy is carried away in gravitational waves.

The new gauge allows simulations to remain accurate far longer than previous standard puncture techniques. Our treatment will generalize, allowing us to study radiation generation in simulations of a variety of initial black hole configurations. Using adaptive mesh refinement, we plan to apply these techniques to study binaries beginning from larger initial separation, which are expected to provide more realistic models corresponding to astrophysical systems. For further understanding of such model dependence, we will compare results from simulations beginning with different initial data models. We will also study the effects of unequal black hole masses, and the individual black hole spins.

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